Multiscale Entanglement Renormalization Ansatz Study of Spin Chains with a Line of Criticality

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Introduction

- Tensor networks are used to describe the properties of quantum systems with local Hamiltonians.
- One such network, the multiscale entanglement renormalization ansatz (MERA)[1], provides a powerful method for simulating quantum systems at a critical point.
- This work investigates spin models which are thought to correspond to $c = 1$ conformal field theories in the thermodynamic limit.

Scale Invariant MERA

- Critical systems are scale invariant, so a scale invariant ansatz is used.
- All layers of tensors are identical (after the first few).
- Green tensors perform Renormalization Group type transformation.
- Blue tensors are unitary disentanglers.
- Each layer represents the system on a different length scale.

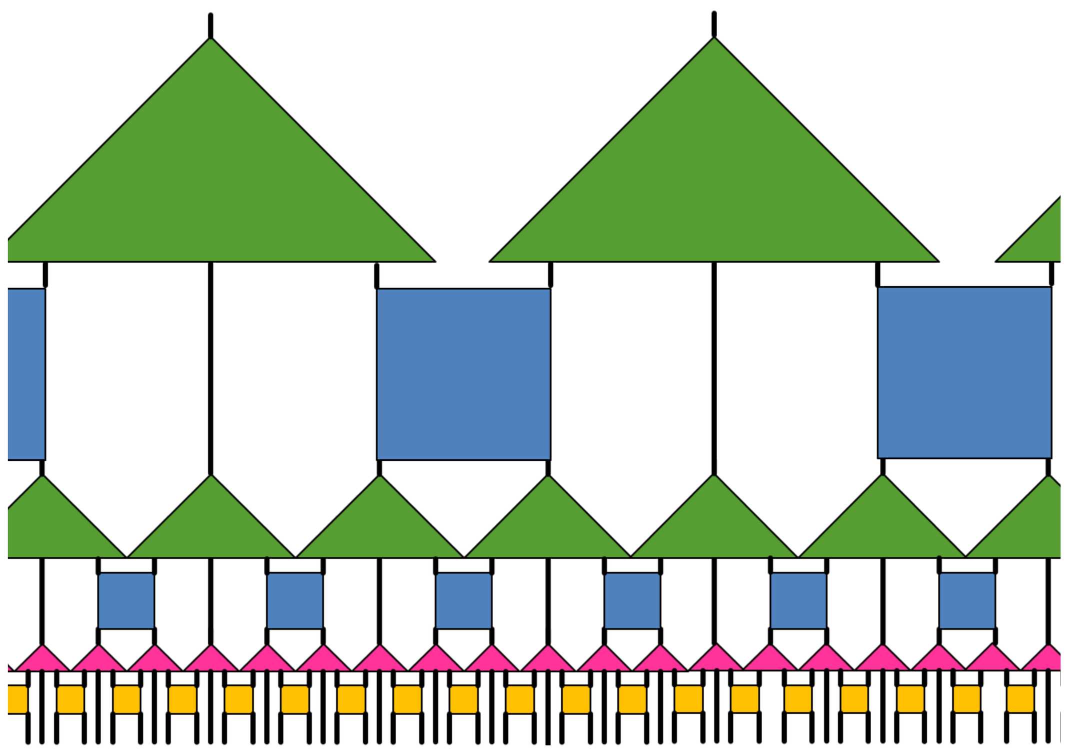


Figure 1: The scale invariant ternary MERA is an ansatz for the wavefunction of critical quantum spin chains.

Two Models to Investigate

- We present two models believed to correspond to $c = 1$ CFTs.

The Ashkin-Teller Model

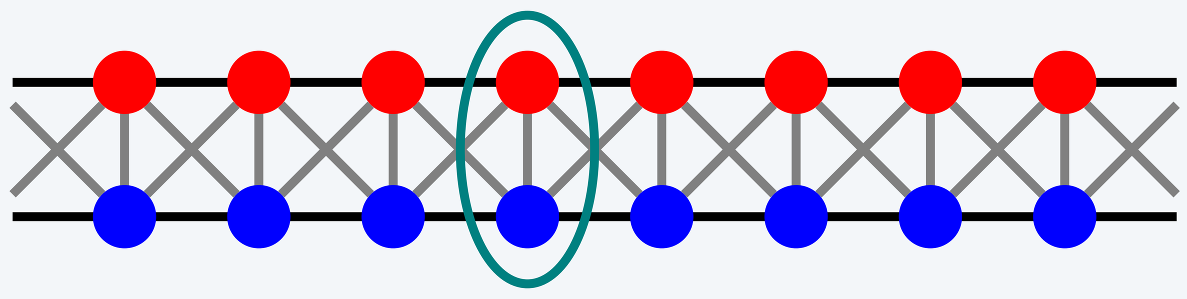


Figure 2: Two coupled Ising chains forming the Ashkin-Teller spin chain. One site is indicated. The model remains critical for a range of λ.

- Two Ising chains coupled by a 4 spin term with strength λ. Described by the Hamiltonian

$$H_{AT} = -\sum_{j=1}^{N} Z_j + Z_j + \lambda Z_j Z_j - \sum_{j=1}^{N-1} \left(X_j X_{j+1} + X_j X_{j+1} + \lambda X_j X_j X_{j+1} X_{j+1} \right). \quad (1)$$

- Remains critical for $\lambda \in [-\sqrt{2}/2, 1]$[2].
- $\mathbb{Z}_2 \otimes \mathbb{Z}_2$ on-site symmetry can be enforced in the MERA.
- Believed to be described by the orbifold boson CFT[3], with radius

$$R_{AT}^2 = \frac{\pi}{2\cos^{-1}(-\lambda)}. \quad (2)$$

The Perturbed Cluster State Model

- Cluster Hamiltonian with $\mathbb{Z}_2 \otimes \mathbb{Z}_2$ symmetry respecting perturbations

$$H_{pCL} = -\sum_{j=1}^{N} X_j + X_j + \lambda X_j X_j - \sum_{j=1}^{N-1} \left(Z_j X_j Z_{j+1} + Z_j X_{j+1} Z_{j+1} + \lambda Z_j Y_j Y Y_{j+1} Z_{j+1} \right). \quad (3)$$

- Believed to be described by the free boson CFT, with radius

$$R_{pCL}^2 = \frac{2}{\pi}(\pi - \cos^{-1}(\lambda)). \quad (4)$$

$c = 1$ CFTs

- The known $c = 1$ CFTs mostly belong to two groups, the compactified free boson (S^1 boson) and orbifold boson.
- CFTs are specified by their *central charge* c, *primary fields* ϕ, their *dimensions* Δ_ϕ and *spin* s_ϕ, and the *OPE coefficients*.
- Exactly marginal primary field leads to continuous variation of the scaling dimensions[5].

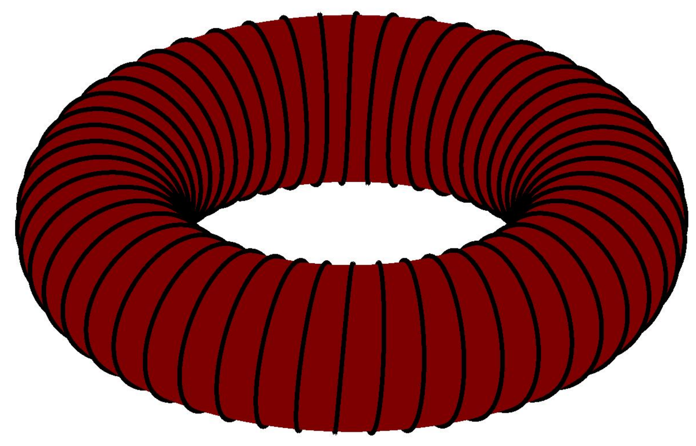


Figure 3: The S^1 boson CFT is the theory of a massless bosonic field φ on a circle.

Conclusions

- We demonstrate the first application of MERA to models with critical lines.
- We show the ability to replicate the expected variation in the conformal data.

References

[1] G. Vidal, Phys. Rev. Lett., **99**, 220405 (2007).

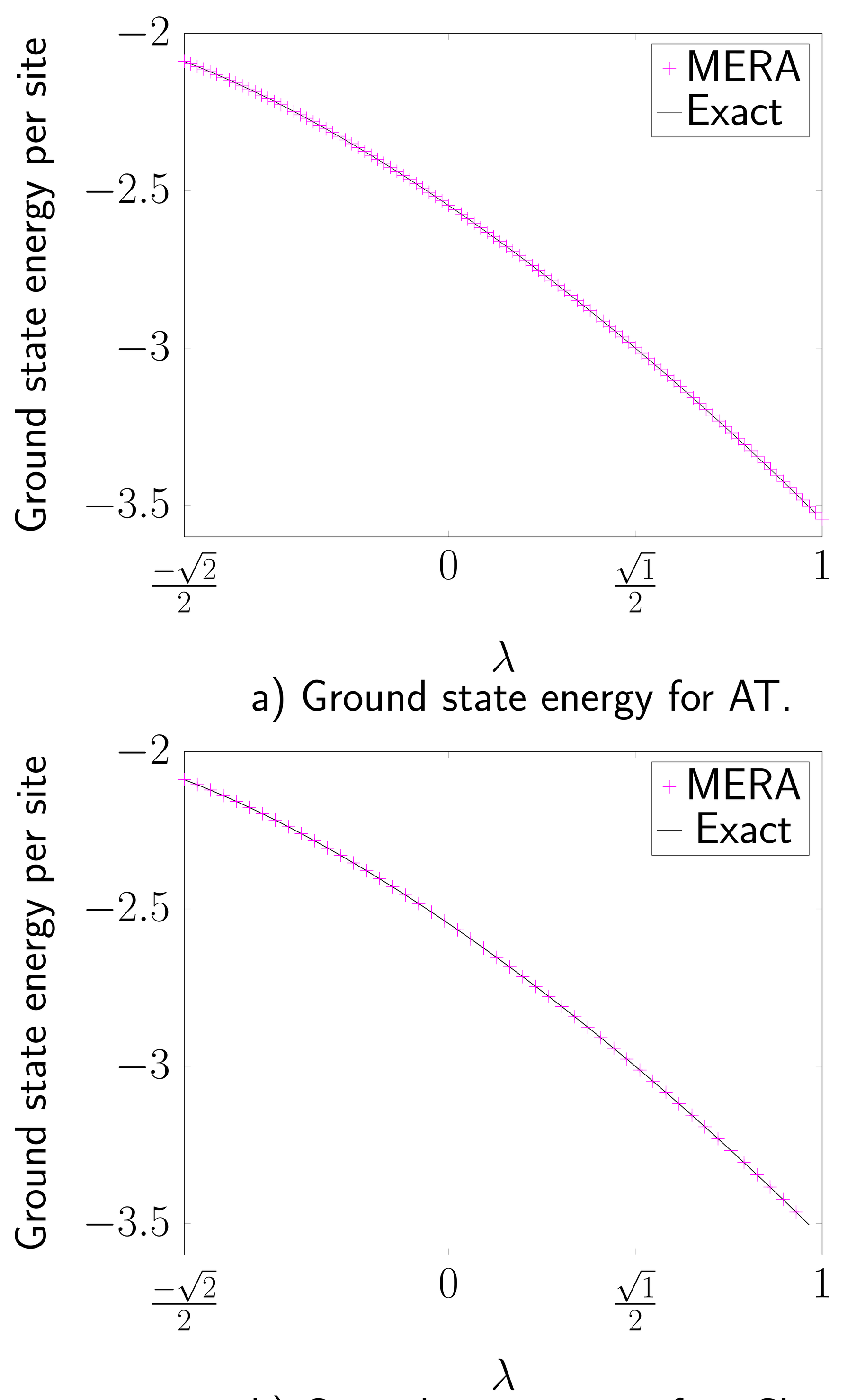
[2] M. Yamanaka et al., Phys. Rev. B, **50**, 559 (1994).

[3] P. Ginsparg, Fields, Strings and Critical Phenomena (Les Houches, Session XLIX) (1988), arXiv:hep-th/9108028.

[4] F.C. Alcaraz et al., Ann. Phys, **182**, 280 (1988).

[5] P. Di Francesco et al., Conformal Field Theory, Springer-Verlag, New York, (1997).

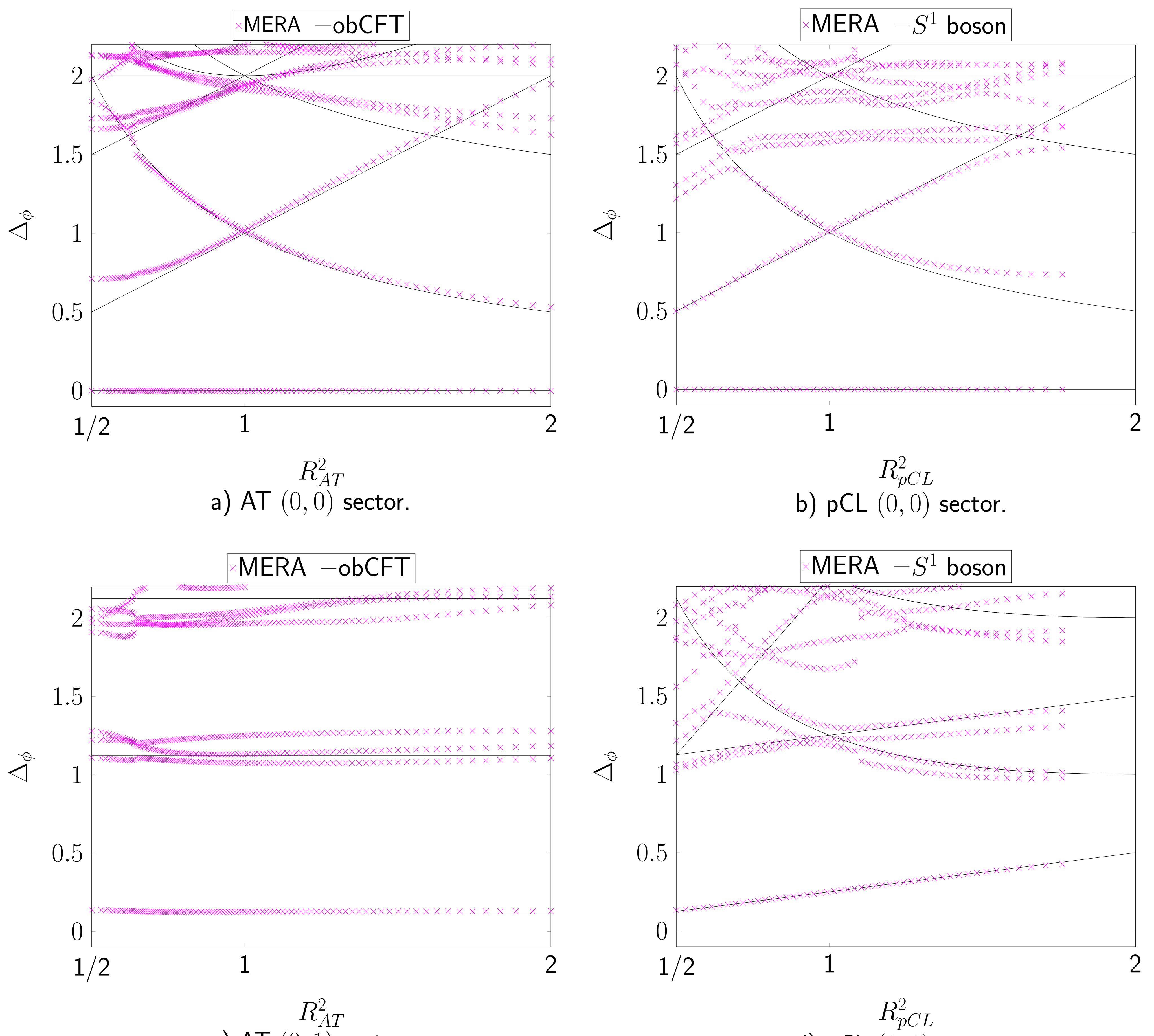
Physical Data Obtained from the MERA



a) Ground state energy for AT.

b) Ground state energy for pCL.

Figure 4: Ground state energies per site (GSE) extracted from the MERA for the Ashkin-Teller (a) and perturbed cluster (b) spin chains. The bond dimensions of the lower and upper indices of the disentangler were $\chi_L = 12$, $\chi_I = 8$ for (a) and $\chi_L = \chi_U = \bar{\chi}/4 = 20$ for (b). The lines marked 'exact' are obtained from numerical integration of the Bethe Ansatz solution to the unitarily equivalent XXZ model[4].



a) AT $(0, 0)$ sector.

b) pCL $(0, 0)$ sector.

c) AT $(0, 1)$ sector.

d) pCL $(0, 1)$ sector.

Figure 5: Scaling dimensions in two of the four symmetry sectors. (a) and (c) are results for the Ashkin-Teller model. Here, $\chi_L = 12$, $\chi_U = 8$ and no projector was used.(b) and (d) show the results in the same sectors for the perturbed cluster model. Here, $\chi_L = \chi_U = 20 = \bar{\chi}/4$.